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Abstract

The many body contributions to the nuclear anapole moment of ^{133}Cs, ^{205}Tl, ^{207,209}PB, and ^{209}Bi from the core polarization are calculated in the random-phase approximation with the effective residual interaction. Strong reduction of a valence nucleon contribution was found provided by the core polarization effects. The contribution of the core particles to the anapole moment compensates this reduction to large extent keeping the magnitude of nuclear anapole moment close to its initial single particle value.

1 Introduction

The atomic parity non-conservation (PNC) effects dependent on nuclear spin are expected to be dominated by contact electromagnetic interaction of electrons with nuclear anapole moment (AM) [1, 2]. The anapole is a new electro-magnetic moment arising in a system without center of inversion [3]. It exists even in such a common object as a chiral molecule in a state with nonvanishing angular momentum [4]. The nuclear anapole moment is induced by PNC nuclear forces.

In all calculations of the anapole moment [2, 5, 6, 7, 8] an independent particle model has been used for the nucleus. In this approach, the AM is determined by the contribution of a single valence nucleon, proton or neutron. The only attempt to account for configuration mixing has been made in [7, 8]. However, as we shall show below, this is only the part of the many-body corrections, and is not the dominant one. The contribution arising from the induced PNC interaction in the nuclear core is considerably larger. It was not discussed yet at all. We present here the first treatment of these kind of effects.

The major coherent effect induced by the residual interaction is the polarization of the nuclear core by a valence nucleon. This is the main effect causing the deviation of nuclear magnetic moments from the Schmidt values. The magnitude of polarization effects depends on the number of transitions from the core states over the Fermi surface. This number is determined by the selection rules, i.e. by the tensor rank of the operator. In the case of the AM, the number of transitions contributing to the core polarization is greater compared to the magnetic moment, and therefore, larger renormalization of the AM is expected. Just to illustrate the above statement, we refer to renormalization of the M3-octupole moment compared to M1. In the case of an octupole moment, the number of transitions over the Fermi surface is much greater, and the core polarization reduces the valence nucleon contribution to M3 by a factor ≈ 4 [9]

A convenient way to describe the core polarization is to use the effective renormalized operators, or effective fields, in the terminology of the theory of finite Fermi systems [10]. In the random phase approximation (RPA), the effective fields are the solutions of a system of integral equations describing particle-hole renormalization of the bare vertex. The weak nucleon-nucleon forces modify these equations. The modifications effectively produce an additional contribution to the AM compensating for the strong reduction of the single particle contribution.

This paper is organized as follows. In the next two sections, we remind the set of operators contributing to the AM, and discuss the accuracy of the leading approximation. Later on, we formulate the basic equations, and introduce the modifications of the equations by the weak nucleon-nucleon interaction in the leading approximation. Next, we calculate the renormalization of the single particle contribution, both analytically and numerically. Finally, we calculate all contributions to the AM both analytically and numerically, and discuss the stability of the results under variation of the constants of the strong residual nucleon-nucleon interaction.

2 The anapole moment operator

The anapole moment operator is defined by [1, 2, 11]

$$\mathbf{a} = -\pi \int d^3r\ r^2 \mathbf{j}(\mathbf{r}), \tag{1}$$

where $\mathbf{j}(\mathbf{r})$ is the electromagnetic current density operator.

The main contribution to the AM comes from the spin part of the current density. Nevertheless, the other contributions are noticeable and, apart from magnetization current, we shall discuss below the contributions from the convection, spin-orbit, and contact currents. Let us define the corresponding parts of the AM in the following way [5]:

$$\begin{aligned}
\mathbf{a}_s^a &= \frac{\pi e \mu_a}{m} \mathbf{r} \times \boldsymbol{\sigma} \\
\mathbf{a}_{conv}^p &= -\frac{\pi e}{m} \left\{ \mathbf{p}, r^2 \right\}; \quad \mathbf{a}_{conv}^n = 0 \\
\mathbf{a}_{ls}^p &= -\pi e U_{ls}^{pn} \rho_0 \frac{N}{A} r^2 \frac{df(r)}{dr} \boldsymbol{\sigma} \times \mathbf{n} \\
\mathbf{a}_{ls}^n &= \pi e U_{ls}^{np} \rho_0 \frac{Z}{A} \frac{d(r^2 f(r))}{dr} \boldsymbol{\sigma} \times \mathbf{n} \\
\mathbf{a}_c^p &= \frac{G}{\sqrt{2}} \frac{\pi e}{m} \rho_0 g_{pn} \frac{N}{A} r^2 f(r) \boldsymbol{\sigma} \\
\mathbf{a}_c^n &= \frac{G}{\sqrt{2}} \frac{\pi e}{m} \rho_0 g_{np} \frac{Z}{A} r^2 f(r) \boldsymbol{\sigma}
\end{aligned} \tag{2}$$

Here $\{\ ,\ \}$ is an anticommutator, ρ_0 is the central nuclear density, $f(r) = \rho(r)/\rho_0$ is the nuclear density profile, and $U_{ls}^{pn} = U_{ls}^{np} = 134\, MeV \cdot fm^5$ is the proton-neutron constant of the effective spin-orbit residual interaction [5]. The contact current contribution arises from velocity dependence of the effective nucleon-nucleon weak forces, taken in the form [5, 11]

$$\begin{aligned}
F_w = \frac{G}{\sqrt{2}} \frac{1}{4m} \sum_{a,b} &\left(\{ (g_{ab} \boldsymbol{\sigma}_a - g_{ba} \boldsymbol{\sigma}_b) \cdot (\mathbf{p}_a - \mathbf{p}_b), \delta(\mathbf{r}_a - \mathbf{r}_b) \} \right. \\
&\left. + g'_{ab} [\boldsymbol{\sigma}_a \times \boldsymbol{\sigma}_b] \cdot \boldsymbol{\nabla} \delta(\mathbf{r}_a - \mathbf{r}_b) \right) = \frac{1}{2} \sum_{ab} F_w(ab).
\end{aligned} \tag{3}$$

Interaction (3) generates a mean field weak potential

$$W_a = \frac{G}{\sqrt{2}} \frac{g_a \rho_0}{2m} \{ \boldsymbol{\sigma} \cdot \mathbf{p}(r) \}, \tag{4}$$

where $g_a = g_{ap} \frac{Z}{A} + g_{an} \frac{N}{A}$.

The effective interaction constants g_{ab}, g_{ba}, g'_{ab} should be, strictly speaking, found from experiment. On the other hand, they can be estimated from the initial finite range PNC-interaction [12] taking zero range limit with the account for short range particle particle repulsion [2, 13]. These "best values" estimates leads to $g_n \ll 1$, while the constant g_p is approximately 4.5. The recent discussion of these constants [14] give, however, different set for g_p and g_n, with $g_p \sim g_n$. Therefore, we shall keep below the constants explicitly as free parameters.

3 Single-particle contribution and leading approximation

The leading approximation for the corrections to the single particle wave functions will be used in calculations of the core polarization effects. Therefore, it is worth to discuss the accuracy of this approximation. Neglecting the spin-orbit potential, and assuming constant nuclear density, we obtain for the correction to the single particle wave function [15]

$$\delta\psi_0(\mathbf{r}) = -\imath\xi_a(\boldsymbol{\sigma}\cdot\mathbf{r})\psi_a(\mathbf{r}), \tag{5}$$

where

$$\xi_a = \frac{G}{\sqrt{2}}g_a\rho_0.$$

Using (5), we obtain for the spin part of the AM

$$\mathbf{a}_s = \frac{Gg\rho_0}{\sqrt{2}}\frac{2\pi e\mu}{m}(R|r^2|R)\frac{K\mathbf{I}}{I(I+1)}, \tag{6}$$

where $K = (l-I)(2I+1)$; R, l and I being the radial wave function, the orbital angular momentum of an outer nucleon and the nuclear spin. It is convenient to discuss AM in terms of a dimensionless constant κ defined as (see Ref.[2])

$$< e\mathbf{a} >= \frac{G}{\sqrt{2}}\frac{K\mathbf{I}}{I(I+1)}\kappa. \tag{7}$$

For the dimensionless constant κ_s we have

$$\kappa_s = \frac{9}{10}g\frac{\alpha\mu}{mr_0}A^{2/3}, \tag{8}$$

where we put $(R|r^2|R) = \frac{3}{5}r_0^2A^{2/3}$.

The naive expression (8) is in rather good agreement with exact numerical calculations of the spin part of AM [2]. Therefore, the approximation is reasonable for averaging the volume type quantities like r^2. For surface type quantities the situation is quite different. An instructive example is the convection current contribution to the AM. Numerical calculation gives for ^{209}Bi $\kappa_{conv} = -0.019$ while in the leading approximation (5) we obtain the value that differs by factor ≈ 5. The explanation consists in the surface nature of the convection current contribution [5]

$$\kappa_{conv} = -\pi g\frac{\alpha\rho_0}{mK}(\delta R|\, r^2\left(\frac{d}{dr}+\frac{K+2}{r}\right)|R). \tag{9}$$

The integrand in the matrix element (9) for the outer nucleon with large angular momentum is peaked at the nuclear surface and the difference between exact $\delta R(r)$ and its approximate expression in leading approximation $rR(r)$ provides considerable changes in the convection current contribution.

4 RPA renormalization of the AM

The AM is a T-odd operator. Thus, the effective two particle interaction involved in AM renormalization must change sign under T-reversal of one of the two particles

$$T_a F(ab) T_a^{-1} = T_b F(ab) T_b^{-1} = -F(ab)$$

The simplest interaction satisfying this condition is the same spin-spin interaction that changes nuclear magnetic moments:

$$F_s(ab) = C\ (g_0 + g_0' \boldsymbol{\tau}_a \cdot \boldsymbol{\tau}_b)\, \boldsymbol{\sigma}_a \cdot \boldsymbol{\sigma}_b \delta(\mathbf{r}_a - \mathbf{r}_b). \tag{10}$$

Here C is the normalization constant that we choose according to [10] $C = 300 MeV \cdot fm^3$ and the constants $g' = 1.01$ and $g = 0.63$.

The effective interaction between the valence and core particles changes the interaction of the valence nucleon, with an external field producing additional core field. In the RPA this effect is accounted for by introducing a dressed effective vertex V satisfying the equation [10]

$$V = V_0([\mathbf{a}_i]) + F\, A\, V. \tag{11}$$

Here $V_0([\mathbf{a}_i])$ is one of the bare AM operators Eq.(2); A is the static polarization loop of a particle-hole pair.

$$A_{\nu_1\nu_1';\nu_2\nu_2'} = \int \frac{d\epsilon}{2\pi\imath}\, G_{\nu_1\nu_2}(\epsilon) G_{\nu_2'\nu_1'}(\epsilon), \tag{12}$$

$G_{\nu_1\nu_2}(\epsilon)$ being a single particle nucleon propagator. In Eq.(11) F is the sum of spin-spin interaction Eq.(10) and the weak effective interaction Eq.(3). The propagator $G_{\nu_1\nu_2}(\epsilon)$ should be calculated in the total mean field potential including the weak potential (4)

It is, however, more convenient to single out the weak interaction effects, treating them explicitly in first order perturbation theory. Let δV be a correction to the vertex from the weak forces. For the unperturbed vertex V and the correction δV we have the equations

$$V = V_0([\mathbf{a}_i]) + F_s\, A\, V, \tag{13}$$

$$\delta V = F_w\, AV + F_s\, \delta AV + F_s\, A\delta V. \tag{14}$$

Here F_w is the weak nucleon-nucleon interaction (3). The AM value is given by

$$a = \langle \delta\psi | V | \psi \rangle + \langle \psi | V | \delta\psi \rangle + \langle \psi | \delta V | \psi \rangle. \tag{15}$$

In the leading approximation, we have

$$a = \imath\xi \langle \psi | [\boldsymbol{\sigma} \cdot \mathbf{r}, V] | \psi \rangle + \langle \psi | \delta V | \psi \rangle. \tag{16}$$

The first term represents the single particle contribution renormalized by the spin-spin interaction, while the second term is an additional contribution from the core particles. Note that the single-particle contribution is now the expectation value of a transformed renormalized anapole operator V. The AM is a T-odd operator of E1 type. The commutator of the AM

with $\imath\boldsymbol{\sigma}\cdot\mathbf{r}$ transforms it, as well as δV, into a T-odd M1 type operator that evidently has nonzero expectation value in a state with spin I. The renormalization effects from the core polarization are different for these two types of operators. In the next two sections we shall discuss the renormalization of the electric type single particle AM operators and the magnetic type operators induced by PNC effects in the core.

Note, that the contact term contribution produces the magnetic type operator from the very beginning. Therefore, its renormalization is similar to that of δV.

5 Renormalization of the electric type single particle operator

To solve Eq.(13), it is convenient to separate the angular dependence, introducing a set of tensor operators of rank J

$$T^L_{JM} = \{\boldsymbol{\sigma}\otimes Y_{Lm}\}_{JM}. \tag{17}$$

The spin part (2) of the AM is proportional to T^1_{1M}. This is the only T-odd operator of the rank 1 with negative parity. Therefore, the dressed vertex V will have the same angular dependence as the bare vertex

$$V_s = v_s(r)\, T^1_{1M}.$$

The core polarization effects the radial dependence only, which for bare spin vertex is just

$$v^{a0}_s(r) = N^a_s\, r,$$

where N^a_s is

$$N^a_s = \imath\sqrt{\frac{8\pi}{3}}\frac{\pi\alpha\mu_a}{m}.$$

The dressed vertexes $v_s(r)$ satisfy the following equations:

$$v^a_s(r) = v^{a0}_s(r) + \sum_{b=p,n} g^{ab}_0 \int_0^\infty r'^2 dr'\; A^b(r,r')v^b_s(r'). \tag{18}$$

Here, the constants g^{pp}_0 and g^{pn}_0 refer to the proton-proton and proton-neutron spin-spin interaction (10) $g^{pp}_0 = g_0 + g'_0$ and $g^{pn}_0 = g_0 - g'_0$. The normalization interaction constant C is included in the radial polarization loop which in our case is

$$A(r,r') = \frac{2}{3}C \sum_{jlnj'l'} k_{jln}\, |\langle jl||T^1_1||j'l'\rangle|^2 R_{jln}(r)R_{jln}(r')G_{j'l'}(r,r';\epsilon_{jln}). \tag{19}$$

Here, k_{jln} are the occupation numbers of filled nuclear states, $\langle jl||T^1_1||j'l'\rangle$ is the reduced matrix element of the tensor operator, and $G_{j'l'}(r,r';\epsilon_{jln})$ is the Green function of the radial Schrödinger equation with the angular momentum j', l' taken at the energy of the occupied level ϵ_{jln}.

Before going over to discussion of the numerical results, let us start from a simple model estimates of the core polarization in a harmonic oscillator potential without spin-orbit interaction. In order to understand orders of magnitude, we shall calculate a polarization loop with the bare spin anapole vertex. Expanding symbolic Eq.(13), we find for the first-order term

$$\mathbf{a}_s^{(1)}(\mathbf{r}) = Cg_s\sigma^j \sum_{\nu\nu'} \psi_\nu^\dagger(\mathbf{r})\sigma^j\psi_{\nu'}(\mathbf{r})\frac{k_\nu - k_{\nu'}}{\epsilon_\nu - \epsilon_{\nu'}}\langle\nu'|\frac{\pi e\mu}{m}(\mathbf{r}\times\boldsymbol{\sigma})|\nu\rangle. \tag{20}$$

Here, $\psi_\nu(\mathbf{r})$ are the single-particle wave functions. In the absence of a spin-orbit potential one can sum over spin variables

$$\mathbf{a}_s^{(1)}(\mathbf{r}) = -Cg_s\frac{\pi e\mu}{m}\boldsymbol{\sigma}\times\sum_{\nu\nu'}\psi_\nu^\dagger(\mathbf{r})\psi_{\nu'}(\mathbf{r})\frac{k_\nu - k_{\nu'}}{\epsilon_\nu - \epsilon_{\nu'}}\langle\nu'|\mathbf{r}|\nu\rangle.$$

In a harmonic oscillator we have a relation

$$\mathbf{r} = -\frac{\imath}{m\omega^2}[H,\mathbf{p}]. \tag{21}$$

Using it, we obtain

$$\begin{aligned}\mathbf{a}_s^{(1)}(\mathbf{r}) &= -\frac{\imath Cg_s}{m\omega^2}\frac{\pi e\mu}{m}\boldsymbol{\sigma}\times\sum_{\nu\nu'}\psi_\nu^\dagger(\mathbf{r})\psi_{\nu'}(\mathbf{r})(k_\nu - k_{\nu'})\langle\nu'|\mathbf{p}|\nu\rangle \\ &= -\frac{\imath Cg_s}{m\omega^2}\frac{\pi e\mu}{m}\boldsymbol{\sigma}\times[\rho(\mathbf{r}),\mathbf{p}] = \frac{Cg_s}{m\omega^2}\frac{\pi e\mu}{m}\boldsymbol{\sigma}\times\boldsymbol{\nabla}\rho(\mathbf{r}).\end{aligned} \tag{22}$$

Taking the expectation value of the correction in the state with total angular momentum $\mathbf{I}$, we find for the ratio of this correction to the zero-order term

$$\begin{aligned}\frac{<\delta\psi|\mathbf{a}_s^{(1)}|\psi>}{<\delta\psi|\mathbf{a}_s|\psi>} &= \frac{<\psi|(\boldsymbol{\sigma}\cdot\mathbf{r})\mathbf{a}_s^{(1)}|\psi>}{<\psi|(\boldsymbol{\sigma}\cdot\mathbf{r})\mathbf{a}_s|\psi>} \\ &= \frac{Cg_s\rho_0}{m\omega^2}\frac{(R|rf'(r)|R)}{(R|r^2|R)} \approx -2.\end{aligned} \tag{23}$$

Since $f'(r)$ is negative, the correction is negative and large. The sign of the correction is defined by the sign of the spin-spin interaction. The repulsive interaction decreases the single-particle contribution. For more realistic potentials, accounting for the spin-orbit potential, we can expect some changes in this ratio, since the correction is maximal on the nuclear surface, where the spin-orbit potential is important. Nevertheless, it remains large.

The results of calculations of the renormalized single particle contribution are listed in Table I and Table II for proton and neutron levels. Note the reduction of the single particle contribution approximately by a factor of 2, in accordance with the above estimates.

The spin-orbit and contact current contributions differ from the previous case only by the radial dependence of their bare vertex. Therefore, their renormalization can be done using the same Eq.(18).

6 Renormalization of the magnetic type operators

Let us now come back to Eq.(14) describing the additional contribution to the AM coming from parity violation effects in the intermediate states of the core particles. Eq.(14) is of the same kind as (13) describing the renormalization of the valence nucleon contribution. The difference is in the driving force or the bare vertex, which is no longer connected to the bare AM operators (2). In calculation of the driving force, we shall use the leading approximation (5).

Expanding the symbolic notation, we obtain

$$F_s(ab)\delta A_b V_b = \delta \sum_{\nu\nu'} \int d^3 r_b \psi^\dagger_\nu(\mathbf{r}_b) F_s(ab) \psi_{\nu'}(\mathbf{r}_b) \frac{k_\nu - k_{\nu'}}{\epsilon_\nu - \epsilon_{\nu'}} \langle \nu' | V | \nu \rangle$$

$$= \imath \xi_b \sum_{\nu\nu'} \int d^3 r_b \left\{ \psi^\dagger_\nu(\mathbf{r}_b) [\boldsymbol{\sigma}_b \cdot \mathbf{r}_b, F_s(ab)] \psi_{\nu'}(\mathbf{r}_b) \langle \nu' | V | \nu \rangle \right.$$

$$\left. + \psi^\dagger_\nu(\mathbf{r}_b) F_s(ab) \psi_{\nu'}(\mathbf{r}_b) \langle \nu' | [\boldsymbol{\sigma} \cdot \mathbf{r}, V] | \nu \rangle \right\} \frac{k_\nu - k_{\nu'}}{\epsilon_\nu - \epsilon_{\nu'}}$$

$$= \imath \xi_b \left\{ [\boldsymbol{\sigma}_b \cdot \mathbf{r}_b, F_s(ab)] \, A_b V_b + F_s(ab) A_b [\boldsymbol{\sigma}_b \cdot \mathbf{r}_b, V_b] \right\}. \tag{24}$$

As we see, the driving force in Eq.(24) consists of two different parts. The first term can be combined with the weak interaction contribution $F_w(ab)A_bV_b$, giving an effective interaction is the sum of the direct and inuced weak interactions

$$\mathrm{F}_w(ab) = F_w(ab) + \imath \xi_b \, [\boldsymbol{\sigma}_b \cdot \mathbf{r}_b, F_s(ab)]. \tag{25}$$

The induced weak interaction was first introduced in [16]. It has the form

$$F_w^{ind}(ab) = \imath [\xi_a \boldsymbol{\sigma}_a \cdot \mathbf{r}_a + \xi_b \boldsymbol{\sigma}_b \cdot \mathbf{r}_b, F_s(ab)], \tag{26}$$

and eventually appears in first order calculations of the residual iteraction. In our case, however, half of interaction (26) enters equation (24). Using the full interaction (26) will produce double counting, because the part related to the valence nucleon is already accounted for in the $\delta\psi$ correction to the valence nucleon wave function. The matrix elements of the induced weak interaction are proportional to the nuclear radius. Therefore, they are enhanced compared to the matrix elements of the direct weak interaction by the factor $A^{1/3}$. For heavy nuclei, this is a considerable factor and, for this reason, we shall omit below the contribution of the direct term.

The second term in (24) produces contributions of a different type to the AM. The commutator $[\boldsymbol{\sigma} \cdot \mathbf{r}, V]$ produces the M1 type of operator, and we can expect its renormalization to be close to that of the magnetic moment. This very contribution has been previously discussed in [7, 8].

According to (24), the correction δV can be presented as a sum of two terms satisfying (24), but with different driving forces

$$\delta V = \delta V^{(1)} + \delta V^{(2)},$$

$$\delta V_a^{(1)} = \imath\xi_b[\boldsymbol{\sigma}_b\cdot\mathbf{r}_b, F_s(ab)]A_bV_b + F_s(ab)A_b\delta V_b^{(1)}$$
$$\delta V_a^{(2)} = \imath\xi_bF_s(ab)A_b[\boldsymbol{\sigma}_b\cdot\mathbf{r}_b, V_b] + F_s(ab)A_b\delta V_b^{(2)}. \tag{27}$$

It is convenient to use, instead of $\delta V^{(2)}$, another variable related to it via

$$\chi_a = \imath\xi_a[\boldsymbol{\sigma}_a\cdot\mathbf{r}_a, V_a] + \delta V_a^{(2)}. \tag{28}$$

This variable satisfies the equation

$$\chi_a = \imath\xi_a[\boldsymbol{\sigma}_a\cdot\mathbf{r}_a, V_a] + F_s(ab)A_b\chi_b. \tag{29}$$

With this definition, we obtain from Eq.(16) the following value for the anapole moment

$$a = \langle\psi|\chi + \delta V^{(1)}|\psi\rangle. \tag{30}$$

Thus, the contributions to the AM can be presented as the expectation value of the sum of two magnetic type operators induced by different driving forces. The first term represents the contribution of the renormalized magnetic operator obtained via Michel transformation [15], while the second is the contribution of the induced weak interaction, similar to that of [16].

Let us now make an analytical estimate of this correction in the model used above for the estimates of the renormalization of single particle AM operator. We shall calculate the driving term in Eq.(27) for $\delta V_a^{(1)}$, using instead of renormalized vertex V the bare spin vertex defined by Eq.(2). The correction to a proton contribution to the AM can be presented in the following form

$$\delta a_s^k \sim \imath\xi_p g_s^{pp}\mu_p\sigma_p^i\sum_{\nu\nu'}\psi_\nu^\dagger(\mathbf{r})[\boldsymbol{\sigma}_p\cdot\mathbf{r}, \sigma_p^i]\psi_{\nu'}(\mathbf{r})\langle\nu'|(\mathbf{r}\times\boldsymbol{\sigma}_p^k)|\nu\rangle\frac{k_\nu^p - k_{\nu'}^p}{\epsilon_\nu - \epsilon_{\nu'}}$$
$$+\,\imath\xi_n g_s^{pn}\mu_n\sigma_p^i\sum_{\nu\nu'}\psi_\nu^\dagger(\mathbf{r})[\boldsymbol{\sigma}_n\cdot\mathbf{r}, \sigma_n^i]\psi_{\nu'}(\mathbf{r})\langle\nu'|(\mathbf{r}\times\boldsymbol{\sigma}_n)^k|\nu\rangle\frac{k_\nu^n - k_{\nu'}^n}{\epsilon_\nu - \epsilon_{\nu'}}. \tag{31}$$

In Eq.(31), we have omitted factors common for protons and neutrons. Calculating the spin commutators we obtain

$$\delta a_s^k \sim \xi_p g_s^{pp}\mu_p\sigma_p^iA_p^{ik}(\mathbf{r}) + \xi_n g_s^{pn}\mu_n\sigma_p^iA_n^{ik}(\mathbf{r}), \tag{32}$$

where

$$A^{ik}(\mathbf{r}) = \sum_{\nu\nu'}\psi_\nu^\dagger(\mathbf{r})(\mathbf{r}\times\boldsymbol{\sigma})^i\psi_{\nu'}(\mathbf{r})\langle\nu'|(\mathbf{r}\times\boldsymbol{\sigma})^k|\nu\rangle\frac{k_\nu - k_{\nu'}}{\epsilon_\nu - \epsilon_{\nu'}}. \tag{33}$$

Summing over the spin variables, we obtain for $A^{ik}(\mathbf{r})$

$$A^{ik}(\mathbf{r}) = \delta_{ik}B^{ll}(\mathbf{r}) - B^{ik}(\mathbf{r}),$$

where

$$B^{ik}(\mathbf{r}) = \sum_{\nu\nu'}\psi_\nu^\dagger(\mathbf{r})r^i\psi_{\nu'}(\mathbf{r})\langle\nu'|r^k|\nu\rangle\frac{k_\nu - k_{\nu'}}{\epsilon_\nu - \epsilon_{\nu'}}. \tag{34}$$

Using again relation (21), we obtain

$$B^{ik}(\mathbf{r}) = \frac{\imath}{m\omega^2} \sum_{\nu\nu'} \psi_\nu^\dagger(\mathbf{r}) r^i \psi_{\nu'}(\mathbf{r}) \langle \nu' | p^k | \nu \rangle (k_\nu - k_{\nu'})$$

$$= \frac{\imath}{m\omega^2} \sum_{\nu} \psi_\nu^\dagger(\mathbf{r}) [r^i, p^k] \psi_\nu(\mathbf{r}) k_\nu = -\frac{1}{m\omega^2} \delta_{ik} \rho(\mathbf{r}), \tag{35}$$

where $\rho(\mathbf{r})$ is the proton or neutron density. Restoring the omitted factors, we have for the correction to the AM of a valence proton

$$\delta \mathbf{a}_s(\mathbf{r}) = -\frac{2\pi e C}{m^2\omega^2} \left(g_s^{pp} \mu_p \xi_p \rho_p(\mathbf{r}) + g_s^{pn} \mu_n \xi_n \rho_n(\mathbf{r}) \right) \boldsymbol{\sigma} \tag{36}$$

Its expectation value in a state with nuclear spin $\mathbf{I}$ is

$$\delta \mathbf{a}_s = \frac{2\pi e C}{m^2\omega^2} \left(g_s^{pp} \mu_p \xi_p \rho_p + g_s^{pn} \mu_n \xi_n \rho_n \right) \frac{K - \frac{1}{2}}{I(I+1)} \mathbf{I}, \tag{37}$$

where K was defined in Eq.(6). For the "best values" of the weak interaction constants [12], $\xi_n \ll \xi_p$ for heavy nuclei. Neglecting ξ_n, we obtain for the ratio of $\delta a_s / a_s$ for a valence proton

$$\frac{\delta a_s}{a_s} = \frac{2 C g_s^{pp} \rho_0}{m\omega^2 (R|r^2|R)} \frac{Z}{A} \left(1 - \frac{1}{2K} \right), \tag{38}$$

where we use the total nuclear density ρ_0 instead of the proton density. Using for ω the standard value $\omega = 41/A^{1/3}$MeV, we find

$$\frac{\delta a_s}{a_s} \approx 4.6 \, \frac{Z}{A}. \tag{39}$$

This result is quite instructive. The contribution to the AM from the induced weak interaction is greater than that coming from the single particle weak potential. However, one should keep in mind that the above calculation has been performed for the bare anapole vertex.

7 Results

The complete results of calculations are summarized in Table I and Table II. In the first column, we list the results of previous AM calculations in the independent particle model. In the second column, we list the contribution of the valence nucleon renormalized by the strong spin-spin interaction. On average, the core polarization reduces the single particle contribution by a factor ≈ 2. In the third column, the AM induced by the core nucleons is written. It significantly restores the reduction of the single particle contribution, leaving the overall renormalization within $\approx 10\%$.

The spin-spin residual interaction depends on the two constants g_0 and g_0' corresponding to the interaction in isoscalar and isovector channels. The major part of the anapole moment is proportional to the nucleon magnetic moments; therefore, the isovector part of the AM

dominates. For this reason, we can expect small sensitivity of the AM to the isoscalar constant g_0. For ^{205}Tl changing g_0 in the interval $0.2 \leq g_0 \leq 0.8$ we get for the AM $0.313 \leq \kappa_{tot} \leq 0.380$. The sensitivity to the isovector constant g_0' is larger, but still within reasonable limits. Changing g_0' in the interval $0.5 \leq g_0' \leq 1.5$ we find for ^{205}Tl $0.450 \geq \kappa_{tot} \geq 0.327$. However, since the constant g_0' is fixed much better from the fit of magnetic moment, the above interval for κ_{tot} is in fact too large.

To summarize, we have calculated the many body contributions to the nuclear anapole moment in the random phase approximation with effective nuclear forces. We found that the contribution to the AM from the valence nucleon is reduced by a factor ≈ 2 from the core polarization effects. However, PNC effects in the core states produce an additional contribution to the AM that partially compensates the reduction of the single particle AM. The resulting value of the AM appears to be close to its initial single-particle value calculated in the independent particle model.

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Table 1: Anapole moment contributions for proton levels

		s.p.	$\langle\delta\psi\|V\|\psi\rangle$	$\langle\psi\|\delta V\|\psi\rangle$	Total
^{133}Cs	κ_s	$0.070g_w^p$	$0.039g_w^p$	$0.018g_w^p+0.009g_w^n$	$0.057\ g_w^p+0.009g_w^n$
	κ_{ls}	$-0.017g_w^p$	$-0.0094g_w^p$	$-0.0035g_w^p-0.0004g_w^n$	$-\ 0.013g_w^p-0.0004g_w^n$
	κ_{conv}	$-0.0041g_w^p$	$-0.0062g_w^p$	$0.0037g_w^p-0.0001g_w^n$	$-\ 0.0025g_w^p-0.0001g_w^n$
	κ_c	$0.0066g_w^{pn}$		$0.0052g_w^{pn}-0.0006g_w^{np}$	$0.0052\ g_w^{pn}-0.0006g_w^{np}$
	κ_{tot}	$0.050g_w^p$ $+0.0066g_w^{pn}$	$0.024g_w^p$	$0.018g_w^p+0.008g_w^n$ $+0.0052g_w^{pn}-0.0006g_w^{np}$	$0.041\ g_w^p+0.008g_w^n$ $+0.0052g_w^{pn}-0.0006g_w^{np}$
^{205}Tl	κ_s	$0.108g_w^p$	$0.052g_w^p$	$0.048g_w^p+0.001g_w^n$	$0.100\ g_w^p+0.001g_w^n$
	κ_{ls}	$-0.022g_w^p$	$-0.011g_w^p$	$-0.009g_w^p-0.0001g_w^n$	$-\ 0.020g_w^p-0.0001g_w^n$
	κ_{conv}	$-0.011g_w^p$	$-0.009g_w^p$	$0.0009g_w^p+0.00002g_w^n$	$-0.0077g_w^p+0.00002g_w^n$
	κ_c	$0.0085g_w^{pn}$		$0.0064g_w^{pn}-0.0006g_w^{np}$	$0.0064\ g_w^{pn}-0.0006g_w^{np}$
	κ_{tot}	$0.075g_w^p$ $+0.0085g_w^{pn}$	$0.032g_w^p$	$0.039g_w^p-0.0003g_w^n$ $+0.0064g_w^{pn}-0.0006g_w^{np}$	$-0.018\ g_w^p-0.0003g_w^n$ $+0.0064g_w^{pn}-0.0006g_w^{np}$
^{209}Bi	κ_s	$0.083g_w^p$	$0.039g_w^p$	$0.032g_w^p+0.0045g_w^n$	$0.071g_w^p+0.0045g_w^n$
	κ_{ls}	$-0.024g_w^p$	$-0.011g_w^p$	$-0.007g_w^p-0.0003g_w^n$	$-0.020g_w^p-0.0003g_w^n$
	κ_{conv}	$-0.005g_w^p$	$0.0009g_w^p$	$0.003g_w^p+0.00004g_w^n$	$0.004g_w^p+0.00004g_w^n$
	κ_c	$0.010g_w^{pn}$		$0.006g_w^{pn}-0.0004g_w^{np}$	$0.006g_w^{pn}-0.0004g_w^{np}$
	κ_{tot}	$0.055g_w^p$ $+0.010g_w^{pn}$	$0.029g_w^p$	$0.028g_w^p+0.004g_w^n$ $+0.006g_w^{pn}-0.0004g_w^{np}$	$0.057\ g_w^p+0.004g_w^n$ $+0.006g_w^{pn}-0.0004g_w^{np}$

Table 2: Anapole moment contributions for neutron levels

		s.p.	$\langle\delta\psi\|V\|\psi\rangle$	$\langle\psi\|\delta V\|\psi\rangle$	Total
^{207}Pb	κ_s	$-0.099g_w^n$	$-0.052g_w^n$	$-0.0035g_w^p-0.028g_w^n$	$-0.0035g_w^p-0.080g_w^n$
	κ_{ls}	$0.006g_w^n$	$0.003g_w^n$	$0.0008g_w^p+0.002g_w^n$	$0.0008\ g_w^p+0.005g_w^n$
	κ_{conv}		$-0.0002g_w^n$	$0.0001g_w^p-0.0002g_w^n$	$0.0001g_w^p-0.0004g_w^n$
	κ_c	$-0.002g_w^{np}$		$0.0001g_w^{pn}-0.001g_w^{np}$	$0.0001\ g_w^{pn}-0.001g_w^{np}$
	κ_{tot}	$-0.093g_w^n$ $-0.002g_w^{np}$	$0.050g_w^n$	$0.003g_w^p-0.027g_w^n$ $+0.0001g_w^{pn}-0.001g_w^{np}$	$-0.003\ g_w^p-0.076g_w^n$ $+0.0001g_w^{pn}-0.001g_w^{np}$
^{209}Pb	κ_s	$-0.062g_w^n$	$-0.033g_w^n$	$-0.004g_w^p-0.033g_w^n$	$-0.004\ g_w^p-0.066g_w^n$
	κ_{ls}	$0.01g_w^n$	$0.004g_w^n$	$0.001g_w^p+0.002g_w^n$	$0.001\ g_w^p+0.006g_w^n$
	κ_{conv}		$-0.0003g_w^n$	$-0.0005g_w^p-0.0002g_w^n$	$-0.0005g_w^p-0.0006g_w^n$
	κ_c	$0.004g_w^{np}$		$0.0003g_w^{pn}-0.003g_w^{np}$	$0.0003\ g_w^{pn}-0.003g_w^{np}$
	κ_{tot}	$-0.052g_w^n$ $0.004g_w^{np}$	$-0.029g_w^n$	$-0.004g_w^p-0.031g_w^n$ $+0.0003g_w^{pn}-0.003g_w^{np}$	$-0.004\ g_w^p-0.060g_w^n$ $+0.0003g_w^{pn}-0.003\ g_w^{np}$